Homework 11

**Definition.** If $f$ is function from $[a,b]$ into $\mathbb{R}$ or $\mathbb{C}$ then the Riemann integral of $f$ with respect to $t$ is

$$
\int_a^b f(t) \, dt = \lim_{\|P\| \to 0} \sum f(t_i^*) \Delta t_i
$$

provided this limit exists. The limit exists, and is equal to the number $I$, if and only if:

- For $\epsilon > 0$,
- There exists $\delta > 0$ so that,
- Whenever $P$ is a partition $a = t_0 < t_1 < \cdots < t_{n-1} < t_n = b$ for which
- $\|P\| = \max \{\Delta t_i : 1 \leq i \leq n\} < \delta$, with $\Delta t_i = t_i - t_{i-1}$.
- Any choice of $t_i^* \in [t_{i-1}, t_i], 1 \leq i \leq n$,
- Gives
  $$
  \left| \sum f(t_i^*) \Delta t_i - I \right| < \epsilon
  $$

**Problem 1.** Assume that $f(t) = u(t) + iv(t)$ with $u, v : [a, b] \to \mathbb{R}$. If $\int_a^b u(t) \, dt$ and $\int_a^b v(t) \, dt$ exist, prove that $\int_a^b f(t) \, dt$ exists and equals $\int_a^b u(t) \, dt + i \int_a^b v(t) \, dt$.

**Problem 2.** Assume that $f : [a, b] \to \mathbb{C}$ is such that $\int_a^b f(t) \, dt$ and $\int_a^b |f(t)| \, dt$ exist. Prove that

$$
\left| \int_a^b f(t) \, dt \right| \leq \int_a^b |f(t)| \, dt
$$