Homework 8

Assume that \( \{a_n\} \) is a bounded sequence of non-negative real numbers with \( \limsup a_n = L \), and that \( \{b_n\} \) is convergent sequence of non-negative reals with \( \lim b_n = B \). Prove the following:

1. \( \forall \epsilon > 0, \exists N \text{ so that } n > N \implies a_n < L + \epsilon \).
2. \( \forall \epsilon > 0, \exists N \text{ so that } n > N \implies a_n b_n < LB + \epsilon \).
3. \( \limsup (a_n b_n) = LB \)

Note: Problem 3 clearly implies Problem 2, if you want to try them in that order.