Homework 8

Assume that $\{a_n\}$ is a bounded sequence of non-negative real numbers with $\limsup a_n = L$, and that $\{b_n\}$ is convergent sequence of non-negative reals with $\lim b_n = B$. Prove the following:

1. $\forall \epsilon > 0$, $\exists N$ so that $n > N \implies a_n < L + \epsilon$.
2. $\forall \epsilon > 0$, $\exists N$ so that $n > N \implies a_n b_n < LB + \epsilon$.
3. $\limsup(a_n b_n) = LB$

Note: Problem 3 clearly implies Problem 2, if you want to try them in that order.