Homework 3

1. Prove that \( f'(c) \) exists if and only if

\[
\lim_{z \to 0} \frac{f(z + c) - f(c)}{z}
\]

exists.

2. Without using Cauchy-Riemann:

   (a) Prove that \( f(z) = 1/z \) is differentiable at every point in its domain.

   (b) Prove that \( f(z) = \bar{z} \) is nowhere differentiable.

   (c) Prove that \( f(z) = |z|^2 \) is differentiable only at \( z = 0 \).

   (d) Determine, with proof, exactly where \( f(z) = \text{Re } z \) is differentiable.