Homework 1

Examine the behavior of each of the following functions $f : \Omega \to \mathbb{C}$,

1. $f(z) = z^2$
2. $f(z) = 1/z$
3. $f(z) = |z|$
4. $f(z) = |z|^2$

If “examine” seems too vague, try some or all of the following.

1. Determine the domain.
2. Find the image of the real axis.
3. Find the image the complex axis.
4. Find the points in the domain that map to real numbers.
5. Find the points in the domain that map to imaginary numbers.
6. Sketch your answers to (4) and (5). The overlap should tell you something about the roots of $f$.
7. Find the image of a square $\{x + iy : 0 \leq x, y \leq c\}$.
8. Find the image of a wedge $\{re^{i\theta} : 0 \leq \theta \leq c\}$