Answers to Worksheet 3

Problem 1: Graphs omitted.

1. There is only one point:
   \[
   \left( \frac{1}{2}, \frac{1}{4} \right)
   \]
   The tangent line is
   \[
   y = \frac{1}{4}
   \]

2. There is only one point:
   \[
   \left( -\frac{1}{2}, -\frac{3}{4} \right)
   \]
   The tangent line is
   \[
   y = 2x + \frac{1}{4}
   \]

3. There are two points:
   \[
   (1 + \sqrt{2}, -2 - \sqrt{2}) \approx (2.414, -3.414)
   \]
   \[
   (1 - \sqrt{2}, -2 + \sqrt{2}) \approx (-0.414, -0.586)
   \]
   The tangent lines are
   \[
   y = (-1 - 2\sqrt{2})x + 3 + 2\sqrt{2} \approx -3.828x + 5.828
   \]
   \[
   y = (-1 + 2\sqrt{2})x + 3 - 2\sqrt{2} \approx 1.828x + 0.172
   \]

Problem 2: Graphs omitted.

1. There is only one point:
   \[
   (0, 2)
   \]
   The tangent line is
   \[
   y = 2
   \]

2. There is only one point:
   \[
   (-\sqrt{2}, \sqrt{2})
   \]
   The tangent line is
   \[
   y = x + 2\sqrt{2}
   \]
3. There are two points:

\((-\sqrt{3}, 1)\)
\((\sqrt{3}, 1)\)

The tangent lines are

\(y = \sqrt{3}x + 4\)
\(y = -\sqrt{3}x + 4\)

**Problem 3:** Graphs omitted.

1. There are infinitely many:

   \[
   \cdots \left(\frac{-3\pi}{2}, 1\right), \left(\frac{-\pi}{2}, -1\right), \left(\frac{\pi}{2}, 1\right), \left(\frac{3\pi}{2}, -1\right), \cdots
   \]

2. There are infinitely many:

   \[
   \cdots (-2\pi, 0), (0, 0), (2\pi, 0), (4\pi, 0), \cdots
   \]

3. There are infinitely many, but the problem only asks for one. I ended up just trying some numbers.

   (a) Look at the graph and guess an answer. I picked \(x = 4.5\) and got

   \((4.5, -0.978)\)

   \(y = -0.211x - 0.029\)

   (b) Refine your guess by trial an error. I decided this was good enough:

   \((4.369, -0.942)\)

   \(y = -0.337x + 0.529\)