SHOW ALL YOUR WORK
This includes any graphs that you obtain from your calculator. If you use a calculator to generate a graph, be sure to sketch a copy of that graph on your exam paper.

1. (10 pts.) Use differentials to find the approximate value of $\sqrt{588}$.

2. (10 pts.) On the interval $[0, 2]$, find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x$$

You must show work that leads to critical points. You may use a graphing calculator after that.

3. (10 pts.) Suppose that $f'(x) < 2$ on the interval $[1, 3]$. If $f(1) = 4$, prove that $f(3) < 8$.

4. (15 pts.) On the interval $[0, \pi/2]$, find all local extrema of

$$f(x) = \cos^2 x + \sin x$$

Be sure to indicate which are minima and which are maxima. You must show work that leads to critical points. You may use a graphing calculator after that.
5. (15 pts.) On what intervals is \( f(x) = x^{1/3} - x^{2/3} \) concave down?

6. (15 pts.) Use the graph of \( f' \) at right and the table of values below to sketch a complete graph of \( f \). Your graph must include all critical points and inflection points and must display the correct concavity on all intervals.

\[
\begin{array}{c|c|c|c|c}
  x & -2 & 0 & 2 & 4 \\
  f(x) & -4 & -3 & 4/3 & 5 \\
\end{array}
\]

7. (10 pts.) Compute \( \lim_{x \to 0^+} x^2 \ln x \)

8. (15 pts.) A rectangle is inscribed in the upper half of the ellipse \( x^2 + 3y^2 = 12 \) as shown at right. What is the largest possible area of such a rectangle?