Homework 5

Ungraded Problems (No graded problems this week)

1. §8.3: Problems 1(a) and (b).

2. §8.3: Problems 1(c) and 3.
   (a) Choose an axis if integration.
   (b) Compute a little bit of force. Express your answer in terms of the integration variable.
   (c) Compute the total force.

3. §8.3: Problem 5. Assume the picture is the end of a tank that extends 30 meters perpendicular to the page.
   (a) Choose an axis of integration.
   (b) Compute a little bit of force on a slice of the semicircular end of the tank.
   (c) Compute the total force on the end of the tank.

4. §8.3: Problem 5. Assume the picture is the end of a tank that extends 30 meters perpendicular to the page.
   (a) Choose an axis of integration.
   (b) Compute a little bit of force on a slice of the curved wall of the tank.
   (c) Compute the total force on the curved wall of the tank.

   Answer: Exactly $3000\rho g$. Approximately $2.94 \times 10^7$ N.

5. §6.3: Problem 15(c) and (d). Be sure to do all the usual steps.

6. §6.3: Problem 33. Assume that the axes have units of meters and that $\rho = 1 \text{ kg/m}^2$.
   NOTE: you could do this problem with formulas from the book, but that will not help you learn how to do the rest of the assignment. I suggest the following outline:

   For $M_x$:

   (a) Chose an axis of integration.
   (b) Draw a typical slice of the shape.
   (c) Compute the moment of that slice (about the $x$-axis).
   (d) Compute the total moment.
For the $y$-coordinate of center of mass:

(a) Sketch a balance axis at some height, say $y = b$.
(b) Chose an axis of integration.
(c) Draw a typical slice of the shape.
(d) Compute the moment of that slice about the balance axis.
(e) Compute the total moment.
(f) Set this equal to zero and solve for $b$.

Answers: $M_x = 4/3g \approx 13.08 \text{ N\cdot m}$.
$M_y = 0 \text{ N\cdot m}$.
Center of Mass: $(0, 2/3)$.

7. §6.3: Problem 33. Axes are in meters and density is variable: $\rho = (y + 1) \text{ kg/m}^2$. Follow the outline above.

Answers: $M_x = 8/3g \approx 26.16 \text{ N\cdot m}$.
$M_y = 0 \text{ N\cdot m}$.
Center of Mass: $(0, 4/5)$.

8. Problem 29. Axes are in inches and density is 0.02 lbs/in$^2$. Find $M_x$, $M_y$ and Center of Mass. Follow the usual outline, but leave answers in integral form if you like.

Answers:

Along $x$-axis, $M_x = \int_0^{\pi/4} 0.02(\cos x - \sin x) \frac{\cos x + \sin x}{2} \, dx = 0.005 \text{ in\cdot lbs.}$

Along $y$-axis, $M_x = \int_0^{1/\sqrt{2}} y \sin^{-1} y \, dy + \int_{1/\sqrt{2}}^{1} y \cos^{-1} y \, dy$

$M_y = \int_0^{\pi/4} 0.02(\cos x - \sin x)x \, dx \approx 0.00221 \text{ in\cdot lbs.}$

Center of Mass : \( \left( \frac{M_y}{\sqrt{2} - 1}, \frac{M_x}{\sqrt{2} - 1} \right) \)

9. Problem 29. Axes are in inches and density is $(0.01 + x) \text{ lbs/in}^2$. Find $M_x$, $M_y$ and Center of Mass. Leave answers as integrals if you wish.