Homework 4

Ungraded Problems

   (a) Graph the curve.
   (b) Choose an axis of integration.
   (c) Compute a tiny bit of length, \( ds \). Express you answer in terms of the integration variable with a pure differential.
   (d) Write down an integral for the length of the curve.

You can check you answers against the answers in the book by having your calculator compute the integrals, perhaps as decimal approximations.

2. §8.1: Go back and compute the exact length for Problems 5 and 7.

3. §8.1: Problem 5 again, with the \( x \)- and \( y \)-axes measured in cm. The curve is a piece of metal whose density varies according to \( \rho = (x + 1) \) g/cm.
   (a) Choose an axis of integration.
   (b) Compute a tiny bit of mass. Express you answer in terms of the integration variable with a pure differential.
   (c) Compute the total mass.

   Answer: About 9.740 g. Exactly \( \frac{105944\sqrt{82} - 806}{98415} \) g.

4. §8.1: Problem 5. \( x \)- and \( y \)-axes measured in cm. Density is \( y \) g/cm.
   (a) Choose an axis of integration.
   (b) Compute a tiny bit of mass. Express you answer in terms of the integration variable with a pure differential.
   (c) Write down an integral for the total mass.

   Answer: \( \int_1^7 \frac{y}{9} \sqrt{81 + \left( \frac{6}{y - 1} \right)^{2/3}} \ dy \)
5. §8.1: Problem 5. $x$- and $y$-axes measured in ft. The curve is a chain that weighs 10 lbs. Write an integral for how much work it takes to lift the entire chain to the height $y = 7$ ft. Show all the usual steps.

Answer: \[ \int_1^7 \frac{7 - y}{9} \sqrt{81 + \left( \frac{6}{y - 1} \right)^{2/3}} \, dy \]

6. §8.1: Problem 5. Starting at the point $(0, 1)$, you walk halfway to the end of the curve. What is your $x$-coordinate?

Answer: About 0.626. Exactly \[ \frac{1}{81} \left( \frac{82\sqrt{82} + 1}{2} \right)^{2/3} - \frac{1}{81} \]

7. §8.1: Problem 5, but assume that $y = 1 + ax^{3/2}$. If the curve is 4 units long, what is $a$?

Note: You will need a calculator or a computer to solve the equation at the end of this problem.

Answer: $a \approx 3.845$.

8. §8.2: Problems 1, 3, 5, 15.

(a) Graph the curve and choose an axis of integration.
(b) Draw and label a typical slice of surface area.
(c) Compute the area of that slice. Express your answer in terms of the integration variable with a pure differential.
(d) Write down an integral for the total surface area.

9. §8.2: Problem 5. Find the surface area.

**Graded Problems**

1. §8.1: Problem 19. Assume $x$- and $y$-axes measured in feet, and the curve has density $y$ lbs/ft.

   (a) Sketch the curve.
   (b) Choose an axis of integration.
   (c) Compute a tiny piece of weight, expressed as a pure differential in the integration variable.
   (d) Write down an integral for the total weight.

2. §8.1: Problem 36(a). Write an integral for the length of the wire. Show all steps, but do not evaluate the integral. If “cosh” is unfamiliar, glance back at §3.9.
3. Evaluate your integral from Problem 2.

4. §8.2: Problem 8.

  (a) Graph the curve and choose an axis of integration.
  (b) Draw and label a typical slice of surface area.
  (c) Compute the area of that slice, expressed as a pure differential in the integration variable.
  (d) Write down an integral for the total surface area.