Homework 3

Ungraded Problems

1. §6.3: Problems 23, 25. Complete the usual four steps, but don’t evaluate the integral.

2. Let \( R \) be the region bounded by \( y = x^2 \) and \( y = \sqrt{x} \). Rotate this region about the \( y \)-axis. Assume \( x \)- and \( y \)-axes have units of cm, and that density varies by \( x \)-coordinate: \( \rho = x + 1 \) g/cm\(^3\) (grams per cubic centimeter).\(^1\)

   (a) Choose an axis of integration.
   (b) Draw and label a typical slice.
   (c) Find the mass of that slice, expressed in terms of the integration variable.
   (d) Find the total mass.

   (Answer: \( 33\pi/70 \) g; approx. 1.481 g)

3. Same shape and rotation axis as in Problem 2. Units on \( x \)- and \( y \)-axes are cm, and density varies by \( y \)-coordinate: \( \rho = y + 1 \) g/cm\(^3\) (grams per cubic centimeter).

   (a) Choose an axis of integration.
   (b) Draw and label a typical slice.
   (c) Find the mass of that slice, expressed in terms of the integration variable.
   (d) Find the total mass.

   (Answer: \( 7\pi/15 \) g; approx. 1.466 g)

4. §6.4: Problems 1, 3, 5.

5. §6.4: Problems 13, 17, 21, 23.

   (a) Choose an axis of integration. Label its origin and positive direction.
   (b) Compute a typical bit of work. Express your answer in terms of the integration variable.
   (c) Compute total work.

6. §6.4: Problem 25. You will have to use a calculator or computer to solve the equation at the end.

   (More on Page 2)

\(^1\)For the technically minded, it’s actually the radial coordinate.
7. §6.4: Problems 7 and 9.

(a) Choose an axis of integration. Label its origin and positive direction.
(b) The rest is up to you.

Graded Problems

1. §6.3: Problem 39.

(a) Choose an axis of integration.
(b) Draw and label a typical slice.
(c) Find the volume of that slice, expressed in terms of the integration variable.
(d) Compute the total volume.

2. Same as Graded Problem 1, but density is $y \text{ g/cm}^3$.

(a) Choose an axis of integration.
(b) Draw and label a typical slice.
(c) Find the mass of that slice, expressed in terms of the integration variable.
(d) Write down an integral for the total mass.


(a) Choose an axis of integration. Label its origin and positive direction.
(b) Finish the problem.

4. §6.4: Problem 16.

(a) Choose an axis of integration. Label its origin and positive direction.
(b) Finish the problem.

5. §6.4: Problem 10. Show all steps. If you don’t know what these are yet, refer to previous problems.