Review for Exam 2

There are three major areas to cover in this exam: Antiderivatives, Numerical Integration, and Infinite Sums, with the last area further divided into continuous and discrete problems.

**Antiderivatives**

- You must memorize the first set of elementary integrals.
- You must be able to recognize and carry out substitutions.
- You should be able to do linear substitutions in your head (especially constant multiple). In particular, you should know elementary integrals 6–9.
- You must be able to recognize and carry out integration by parts.
- You must be able to recognize and carry out partial fractions decomposition with linear denominators.
- You should be able to recognize when an antiderivative is not one of these three types, but for this exam any such problem will come with a suggested substitution, a useful trig identity, or some other revealing hint.
- Any of the above antiderivatives could show up in a definite integral or an improper integral.

**Numerical Integration**

- You should know how to compute Left, Right, Midpoint, Trapezoid and Simpson’s approximations. The formula for Simpson’s Rule will be provided.
- You should know how to compare Left, Right, Midpoint and Trapezoid approximations to each other and to the true value of the integral.
- You should be able to compute the error in a Midpoint, Trapezoid or Simpson’s approximation. Formulas will be provided. If $f''$ or $f^{(4)}$ is too complicated, then a graph of the derivative will be provided.
- You should be able to compute the number of intervals required to meet a specified error bound. If $f''$ or $f^{(4)}$ is too complicated, then a graph of the derivative will be provided.
- You should be able to approximate an integral to within a specified error bound. The process is:
  1. Choose a method.
  2. Determine a number of intervals.
  3. Compute the approximation.
Infinite Sums (Integrals and Series)

- You must be able to identify an integral or series as convergent or divergent. The basic rule is "big denominators cause little fractions, which cause convergence."

- You need to know what’s "big" and what’s "small". Here's a partial list from small to large. (These are discrete functions, but the rules are the same for continuous functions, just replace \( n \) with \( x \).)

\[
\ln n < \cdots < n < n^2 < n^3 < \cdots < 2^n < e^n < 3^n < \cdots < n!
\]

- You must be able to create a strong comparison. That is, find a new function that is
  1. bigger than the old one.
  2. nice enough to antidifferentiate.
  3. still convergent.

- You must be able to use a strong comparison to bound the tail of an integral or a series. There are three possible situations, depending on whether or not the original \( f(x) \) is nice enough to integrate.

\[
\begin{align*}
1. \quad & \int_a^\infty f(x) \, dx \leq \int_a^\infty g(x) \, dx \\
2. \quad & \sum_{a}^{\infty} f(n) \leq \int_{a-1}^{\infty} f(x) \, dx \\
3. \quad & \sum_{a}^{\infty} f(n) \leq \int_{a-1}^{\infty} g(x) \, dx
\end{align*}
\]

- Know how to compute the (approximate) value of an infinite integral or series. The process is
  1. Pick a cutoff point.
  2. Use strong comparison to compute the error in the tail.
  3. Compute the non-tail part. Use numerical integration for integrals; use addition for series.
  4. (For integrals only) Compute the error in your numerical method.

- Be able to approximate an infinite integral or sum to with a specified accuracy. The process is:
  1. Use a strong comparison to bound the tail.
  2. Use your error bound (or part of it) to solve for a cutoff point.
  3. (For integrals only) Use the remaining part of your error bound to solve for the number of intervals needed.
  4. Compute the non-tail part. Use numerical integration for integrals; use addition for series.