Review for Exam 1

This is an exam about integration. I.e., adding stuff up. Most of the problems will be about what to add up, rather than how to add. That means mostly set up, and not much integration.

Overview

- Expect to see about six or seven problems that look very much like homework problems.
- Expect one or two more that do not look like previous homework. These will be problems that ask you to add up something that we haven’t done in the homework.
- You will have 50 minutes for the exam, so, on average, you will have to work a problem every 6.25 minutes. Keep that in mind when you are studying.
- There will be at most two problems where you actually have to integrate.
- Be prepared for at least one substitution.

Geometry

All integration problems have a geometric basis. Here is a list of all the shapes that you could possibly encounter. In all cases, you must be able to write these expressions completely in terms of the integration variable. (In these examples, always \( x \).)

- One dimensional, straight:
  \[ dx \]

- One dimensional, curved:
  \[ ds = \sqrt{(dx)^2 + (dy)^2} \]

- Two dimensional, flat:
  \[ dA = h \, dx \]

- Two dimensional, surface of rotation:
  \[ dA = 2\pi r \, ds \]

- Three dimensional, volume of rotation:
  \[ \text{slice } x\text{-axis: } dV = \pi [R^2 - r^2] \, dx \]
  \[ \text{slice radial axis: } dV = 2\pi rh \, dx \]

- Three dimensional, cross-sections have area \( A \):
  \[ dV = A \, dx \]
Extras
The geometry gets you a little bit of stuff. Sometimes you just add up the amount of stuff (length, area, or volume problems). Other times you have to do something with the stuff. Here’s a list of what we have done so far, organized by geometry.

- If the stuff is $dx$:
  1. Lift it (compute work).
  2. Exert force across the distance $dx$.
  3. If $x$ is time, compute distance as rate $\times$ time.

- If the stuff is $ds$:
  1. Find its mass or weight.
  2. Lift it (compute work).

- If the stuff is $dA$ (flat area):
  1. Find its value (as property, say).
  2. Find its mass or weight.
  3. Compute force (applied by fluid pressure).
  4. Compute its moment(s).

- If the stuff is $dA$ (curved area):
  1. Nothing except find total area.
  2. But I could certainly think of a thing of two!

- If the stuff is $dV$ (rotated shape):
  1. Find its mass or weight.

- If the stuff is $dV$ (cross-sections):
  1. Lift it (compute work).

NOTE: There is no expectation that you know any physics. You do not need to memorize any formulas. I will supply any that do not seem obvious. If you think you need a physics formula that I did not include, just ask during the exam.

Harder Stuff
- Any of these problems could involve unknown constants.
- You might have to apply a physical constraint to solve for an unknown constant. This is the time you are most likely to have to actually compute an integral.
• Any of the “extras” applications could show up attached to any of the geometric shapes. For instance: volume, but compute the moment; or curved area, but compute total mass. The possibilities are nearly endless. The idea is that you should know how to *add stuff up*. No matter what the stuff.

• For that matter, I can (and probably will) dream up some totally new thing to add up.

• However, there will NOT be any new geometry.

**Study Tips**

• Work homework!

• You goal should be to do homework to the point that you can look at a problem and immediately know what to do with it.

• Once you know what to do, you should be able to do it quickly.

• The only way to get to this point is to work a large number of homework problems. If you feel that I did not assign enough, work some more. Look for unassigned problems that are similar to the assigned ones. If you need more: do even ones; make up your own densities; pick any length, area, or volume problem and lift it.