1. (10 pts.) How many strings of three distinct capital letters include the letter ”A”?

Solution:

There are three positions in which the ”A” can occur.
There are 25 · 24 ways to fill in the other two positions.

\[ 3 \cdot 25 \cdot 24 = 1,800 \]

2. (10 pts.) How many permutations of the digits 1–9 do not contain the sequence 123?

Solution:

Count the number in which the sequence 123 does appear.

7 possible places for 123
6! choices for the remaining digits
7 \cdot 6! = 7!

Subtract from the total number of permutations

\[ 9! - 7! = 317,520 \]
3. (10 pts.) A coed softball team consists of 5 women and 5 men. How many teams can be made from a group of 15 men and 12 women?

Solution:

From 15, choose 5; from 12 chose 5.

\[
\binom{15}{5} \binom{12}{5} = \frac{15!}{5!10!} \cdot \frac{12!}{5!7!} \\
= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{5 \cdot 4 \cdot 3 \cdot 2} \cdot \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2} \\
= (7 \cdot 13 \cdot 3 \cdot 11)(11 \cdot 9 \cdot 8) \\
= 2,378,376
\]