1. (10 pts.) Of 30 people who ate at a restaurant, 12 had steak, 15 had wine, and 11 had dessert. If 6 had both steak and wine, 10 had both wine and dessert, 5 had both steak and dessert, and 4 had all three, how many had none of these?

Solution:

Let $S$ be the set of people who had steak.
Let $W$ be the set of people who had wine.
Let $D$ be the set of people who had dessert.
We need to count the people in the set $(S \cup W \cup D)^C$.

$$|(S \cup W \cup D)^C| = 30 - |S \cup W \cup D|$$
$$= 30 - [|S| + |W| + |D| - |S \cap W| - |S \cap D| - |W \cap D| + |S \cap W \cap D|]$$
$$= 30 - [12 + 15 + 11 - 6 - 10 - 5 + 4]$$
$$= 9$$
2. (20 pts.) Find the number of integers between 1 and 200 (including 1 and 200) that are:

(a) Divisible by at least one of the integers 4, 5 and 6.
(b) Divisible by at least two of the integers 4, 5 and 6.

Solution:

Let $A$ be the set of integers divisible by 4.
Let $B$ be the set of integers divisible by 5.
Let $C$ be the set of integers divisible by 6.
$A \cap B$ is the set of integers divisible by 20.
$A \cap C$ is the set of integers divisible by 12 ($\text{lcm}$ of 4 and 6).
$B \cap C$ is the set of integers divisible by 30.
$A \cap B \cap C$ is the set of integers divisible by 60 ($\text{lcm}$ of 4, 5, and 6).

(a) We need to count $A \cup B \cup C$.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= \frac{200}{4} + \frac{200}{5} + \left[ \frac{200}{6} \right] - \frac{200}{20} - \left[ \frac{200}{12} \right] - \frac{200}{30} + \left[ \frac{200}{60} \right]$$

$$= 50 + 40 + 33 - 10 - 16 - 6 + 3$$

$$= 94$$

(b) We need to count $A \cap B$ and $A \cap C$ and $B \cap C$, without triple counting the overlap. So...

$$|A \cap B| + |A \cap C| + |B \cap C| - 2 \cdot |A \cap B \cap C| = 10 + 16 + 6 - 2 \cdot 3$$

$$= 26$$