Hw 7 Solutions

Proposition 4.4(a): Every angle has a unique bisector.

**Note:** You do not have a definition of angle bisector. The proof below conveys what I think is a working definition. Yours should do so as well.

**Proof:** Given $\angle ABC$.

Find $C'$ on $AC$ so that $AB \cong AC'$.

Let $D$ be the midpoint of $AC'$.

Be definition of midpoint, $AD \cong DC'$.

By construction, $BA \cong BC'$.

$BD \cong BD$.

By SSS, $\triangle DBA \cong \triangle DBC'$.

We now have $\overrightarrow{BD}$ inside $\angle ABC$ and $\angle ABD \cong \angle CBD$.

Therefore $\overrightarrow{BD}$ bisects $\angle ABC$.

Now suppose that $\overrightarrow{BE}$ also bisects $\angle ABC$.

$\overrightarrow{BE}$ is inside $\angle ABC$.

$\therefore \overrightarrow{BE}$ meets $AC'$ in a point $E'$.

$BA \cong BC'$.

$\angle EBA \cong \angle EBC'$ by definition of bisector.

$BE \cong BE$.

By SAS, $\triangle EBA \cong \triangle EBC'$.

$\therefore AE \cong EC'$.

This makes $E$ a midpoint of $AC$.

By uniqueness of midpoints, $E = D$ ■
**Proposition 4.4(b):** Every segment has a unique perpendicular bisector.

**Note:** As above, you will have to convey an appropriate definition of perpendicular bisector.

**Proof:** A segment $AB$ has a unique midpoint $M$.
There is a line $l$ through $M$ with $l \perp AB$.
Therefore $l$ is a perpendicular bisector of $AB$.
Let $k$ be another perpendicular bisector of $AB$.
By definition of bisector, $k$ meets $AB$ at $M$.
$k$ forms a right angle on each side of $AB$ with vertex $M$.
So does $l$.
All right angles are congruent, so
by uniqueness of angle construction (Axiom C4), $l = k$ ■

**Proposition 4.11:** Hilbert’s parallel postulate implies the angle sum of any triangle is $(180)^\circ$.

**Proof:** Given $\triangle ABC$.
Find $D$ so that $\angle DBA \cong \angle A$
with $D$ and $C$ on opposite sides of $AB$.
$\overrightarrow{BA}$ is inside $\angle DBC$.
:. $(\angle DBA)^\circ + (\angle ABC)^\circ = (\angle DBC)^\circ$.
It follows that $(\angle A)^\circ + (\angle ABC)^\circ = (\angle DBC)^\circ$.
Now find $E$ so that $\angle EBC \cong \angle C$
with $E$ and $D$ on opposite sides of $BC$.
By congruence of alternate interior angles,
$\overrightarrow{DB} \parallel \overrightarrow{AC}$ and $\overrightarrow{EB} \parallel \overrightarrow{AC}$.
Hilbert’s postulate says $\overrightarrow{DB}=\overrightarrow{EB}$.
Since $D$ and $E$ are on opposite sides of $BC$,
$\angle DBC$ and $\angle EBC$ are supplementary.
:. $(\angle DBC)^\circ + (\angle EBC)^\circ = (180)^\circ$.
By substitution, $(\angle A)^\circ + (\angle ABC)^\circ + (\angle C)^\circ = (180)^\circ$ ■