Hw 11 Solutions

Exercise 2. Suppose that lines $l$ and $l'$ have a common perpendicular segment $MM'$.
Let $A$ and $B$ be points on $l$ such that $M$ is not the midpoint of segment $AB$. Prove that $A$ and $B$ are not equidistant from $l'$.

Proof: Suppose that they are equidistant from $l'$.
Drop $AA' \perp l'$ and $BB' \perp l'$.
$\Box A'B'B'A$ is a Saccheri quadrilateral.
Let $PP'$ be the segment joining the midpoints of $AB$ and $A'B'$.
By Lemma 6.2, $PP' \perp l$ and $PP' \perp l'$.
But $MM'$ is the only such segment.
$\therefore M = P$.
However, $M$ is not the midpoint of $AB$.

Exercise 4. Given the figure at right. Prove that $AA' < BB'$.

Proof: $\Box M'A'AM$ is a Lambert quadrilateral.
It cannot be a rectangle, so $\angle MAA' < 90$.
The supplementary angle is $\angle BAA'$.
$\therefore \angle BAA'$ is obtuse.
$\Box M'B'B'M$ is a Lambert quadrilateral.
$\therefore \angle MBB' < 90$.
We now have $\angle BAA' > \angle ABB'$.
$\therefore BB' > AA'$.