Axiom System for Exam 1

Definitions

D1: Given distinct points $A$ and $B$, the segment $AB = \{P \mid A \ast P \ast B\} \cup \{A, B\}$.  

D2: Given distinct points $A$ and $B$, the ray $\overrightarrow{AB} = AB \cup \{P \mid A \ast B \ast P\}$.  

D3: Let $l$ be a line and let $A$ and $B$ be points not on $l$. If $A = B$ or if $AB$ does not meet $l$, then $A$ and $B$ are on the same side of $l$. 

D4: Let $l$ be a line and let $A$ and $B$ be points not on $l$. If $A \neq B$ and $AB$ does meet $l$, then $A$ and $B$ are on opposite sides of $l$. 

D5: Given an angle $\angle CAB$, a point $D$ is in the interior of $\angle CAB$ if $D$ and $B$ are on the same side of $AC$ and if $D$ and $C$ are on the same side of $AB$. 

Axioms

I1: For every point $P$ and for every point $Q \neq P$ there exists a unique line $l$ incident with $P$ and $Q$.  

I2: For every line $l$ there exist at least two distinct points incident with $l$. 

I3: There exists three distinct, non-collinear points. 

B1: If $A \ast B \ast C$, then $A$, $B$ and $C$ are distinct, collinear points, and $C \ast B \ast A$. 

B2: Given two distinct points $B$ and $D$, there exist points $A$, $C$ and $E$ lying on $\overrightarrow{BD}$ such that $A \ast B \ast D$, $B \ast C \ast D$, and $B \ast D \ast E$. 

B3: If $A$, $B$ and $C$ are distinct collinear points, then exactly one of $A \ast B \ast C$, $A \ast C \ast B$, and $C \ast A \ast B$ is true. 

B4: For every line $l$ and for any three points $A$, $B$ and $C$ not on $l$:

(i) If $A$ and $B$ are on the same side of $l$ and $B$ and $C$ are on the same side of $l$, then $A$ and $C$ are on the same side of $l$. 

(ii) If $A$ and $B$ are on opposite sides of $l$ and $B$ and $C$ are on opposite sides of $l$, then $A$ and $C$ are on the same side of $l$. 

(iii) If $A$ and $B$ are on opposite sides of $l$ and $B$ and $C$ are on the same side of $l$, then $A$ and $C$ are on opposite sides of $l$. 

1
Propositions

Proposition 2.5: For every point $P$ there exist at least two lines through $P$.

Proposition 3.3: Given $A \ast B \ast C$ and $A \ast C \ast D$, then $B \ast C \ast D$ and $A \ast B \ast D$.

Corollary: Given $A \ast B \ast C$ and $B \ast C \ast D$, then $A \ast B \ast D$ and $A \ast C \ast D$. 