Review for Exam 1

What follows is intended as a guide to focus your study for Exam 1. Read the Overview and the Study Tips. After that there is list, arranged by section, of the ideas and techniques that you must know for the exam.

Overview

- Expect to see about ten (10) problems that look very much like homework problems.
- You will have 50 minutes for the exam, so, on average, you will have to work a problem every 5 minutes.
- About 50% to 60% of the exam will be problems that should remind you of the easy to moderate homework problems.
- Another 30% to 40% will look like the more involved homework problems.
- The last 10% will be a new problem that requires an original application of the ideas and/or techniques of Chapters 1 and 2.

Study Tips

- Do lots of homework!
- Your goal should be to do so much homework that you can look at a problem and immediately know what to do with it.
- Once you know what to do, you should be able to do it quickly.
- The only way to get to this point is to work a large number of homework problems. If you feel that I did not assign enough, work some more. Look for unassigned problems that are similar to the assigned ones. If you need more, do even ones. If you need more than that, go to the review problems at the end of each chapter.

Section 1.1

- Know that a function is:
  1. A set of inputs (the domain.)
  2. A rule for generating outputs.
- Know how to evaluate a function given a formula, a graph, or a table of values.
- Know how to find the domain of a function given by a formula, a graph, or a table of values.
- Be able to create a formula from a description of a function. I.e. know how to work the Section 1.1 word problems.
Section 1.2

- As above, be able to create a formula from a description of a function.

Section 1.3

- Given a graph of a function \( f(x) \) and a number \( a \), be able to find the graph and domain of \( f(x) + a \), \( af(x) \), \( f(x + a) \), \( f(ax) \), and any combination of these transformations.
- Conversely, given a graph of a transformed function, be able to write a formula for it.
- Given functions \( f \) and \( g \), be able to evaluate, graph, and/or find the domain of \( f + g \), \( f - g \), \( fg \), \( f/g \), and any other algebraic combination of these.
- Be able to do this if the functions are given as formulas, graphs, or tables.
- Be able to evaluate, graph, and/or find the domain of the composition of two functions.
- Be able to do this if the functions are given as formulas, graphs, or tables.

Section 1.5

- Know the domains, range and graph of any exponential function, \( f(x) = b^x \).
- In particular, know the graphs of \( f(x) = e^x \) and \( f(x) = e^{-x} \).
- Given points on a graph of \( f(x) = Ab^x \), be able to solve for \( A \) and \( b \).

Section 1.6

- Be able to determine if a function is one-to-one.
- Be able to do this if the function is given by a formula, a graph, or a table of values.
- Know the basic rule that relates a function and its inverse:

\[
f(x) = y \iff f^{-1}(y) = x
\]

- Know that this means “Inputs and outputs trade places.”
- Given a function \( f \) (either by formula, graph, or table) be able to compute \( f^{-1} \) of a single input.
- Given a function described by a formula, be able to construct a formula for \( f^{-1} \).
- Given a graph of a function, be able to graph the inverse function.
- Know the definitions of \( \log_b x \) and \( \ln x \). I.e.

\[
\log_b x = y \iff b^y = x \\
\ln x = y \iff e^y = x
\]
• Know the domain, range and graph of any logarithmic function, \( \log_b x \). In particular, know the domain, range and graph of \( \ln x \).

• Know how to solve equations in which the variable is an exponent.

• Know how to solve equations in which the variable is inside a logarithm.

Section 2.1

• Be able to compute secant slopes and average velocities.

• Be able to use your computations to guess a tangent slope or an instantaneous velocity.

• Be able to do this if the function is given by a formula, a graph, or a table of values.

Section 2.2

• Be able to compute limits and/or one sided limits by looking at a graph.

• Be able to compute limits and/or one sided limits by plugging in inputs near the limit point and then making a reasonable guess.

• Know when a limit does not exist and be able to explain why.

• Be able to identify infinite limits (vertical asymptotes) of a function. Be able to do this if the function is given by a formula or a graph.

Section 2.3

• Given some functions whose limits are known, be able to compute limits of any algebraic combination of those functions.

• Be able to do this if the functions are given by formulas, graphs, or tables.

• Know when you can take a limit by just plugging a number into a function.

• Know how to take a limit by simplifying algebraically until you can plug in the limit point. (I.e., factor and cancel!)

• Know how to take a limit by squeezing.

• Know how to take limits of piecewise defined functions and functions involving absolute value.

Section 2.5

• Know the definition of continuity at a point.

• Be able to identify whether a function is or is not continuous a particular point.

• Be able identify the intervals on which a function is continuous.
Section 2.6

- Be able to compute limits of functions at $\pm \infty$.
- Be able to do this if the function is given by a formula or a graph.
- Be able to compute limits at $\pm \infty$ by squeezing.

Section 2.7

- Be able to compute secant slopes and average rates of change (including velocities).
- Be able compute tangent slopes and instantaneous rates of change by taking limits.

Section 2.8

- Be able to compute the derivative of a function at a point.
- Note that if the function is given by a graph or a table of values, you will have to resort to the “plug in and guess” method of Sections 2.1 and 2.2.
- If the function is expressed algebraically use your favorite one of:

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- Know what the derivative at a point measures. I.e.

1. $f'(a)$ is the slope of the tangent line to $f(x)$ at the point $(a, f(a))$.
2. $f'(a)$ is the rate of change of $f(x)$ with respect to $x$ when $x = a$.

Section 2.9

- Be able to find the derivative of a function and express your answer as a function.
- Given a graph of a function, be able to sketch the graph of its derivative.