Review for Exam 2

What follows is intended as a guide to focus your study for Exam 2. Read the Overview and the Study Tips. After that there is list, arranged by section, of the ideas and techniques that you must know for the exam.

Overview

- Expect to see somewhere between 10 to 12 problems that look very much like homework problems.
- About 50% to 60% of the exam will be problems that should remind you of the easy to moderate homework problems.
- Another 30% to 40% will look like the more involved homework problems.
- The last 10% will be a new problem that requires an original application of the ideas and/or techniques from Chapters 3 and 4.
- VERY IMPORTANT NOTE: A graphing calculator is not required for this test, but there are several places where one could be used. If you use your calculator to draw a graph, please “show your work” by sketching a copy of the calculator’s graph on your test paper.

Study Tips

- Do lots of homework!
- You goal should be to do so much homework that you can look at a problem and immediately know what to do with it.
- Once you know what to do, you should be able to do it quickly.
- The only way to get to this point is to work a large number of homework problems. If you feel that I did not assign enough, work some more. Look for unassigned problems that are similar to the assigned ones. If you need more, do even ones. If you need more than that, go to the review problems at the end of each chapter.

Section 3.1

- Be able tell at a glance what the end behavior of a polynomial will be.
- Know that roots correspond to factors of a polynomial.
- Be able to tell at a glance (provided it’s factored) how a polynomial behaves near any root.
- Be able to use this information to quickly sketch a graph of a polynomial, provided it’s factored.
- Be able to work word problems from this section. Pay particular attention to domains.
Section 3.2

- Be able to do long division.
- Be smart enough NOT to do long division when the divisor is linear.
- For linear divisors, be able to:
  1. Find quotient and remainder.
  2. Find output of a polynomial. (It’s just the remainder.)
  3. Determine if a divisor is a factor.
  4. Determine if a number is a root.
- Given roots of a polynomial, be able to find the polynomial.
- Be able to solve such a problem with an additional condition, such as integer coefficients, constant term 1, etc.

Section 3.3

- Know the basic routine for finding the roots of a high degree polynomial:
  1. Guess a root.
  2. Check by division (the short way).
  3. Repeat until you get down to a quadratic.

NOTE: On these problems you must show work in which you check you guesses. You must also show me a picture of anything you get from a graphing calculator.

- Know the basic routine for factoring a high degree polynomial:
  1. Find the roots.
  2. Turn them into factors.

Section 3.4

- Know what a complex number is.
- Know that \(i^2 = -1\).
- Be able to add, subtract, multiply and divide complex numbers. Note that your answers must be in the form \(a + bi\).

  NOTE: Most calculators will do this. I need you to do it by hand and show your work.
- Be able to find complex roots of quadratic equations.
Section 3.5

- Be able to do the Section 3.3 problems, but with complex roots.
- Know that a real coefficient polynomial can only have complex roots in conjugate pairs.

Section 3.6

You will have an exam problem that asks you to graph a rational function. Your calculator might be of some help, but you need to show your work as well, and you need some specific details in the graph.

- You must show work that finds the roots. Factoring is sufficient.
- You must show work that finds the vertical asymptotes. Factoring is sufficient.
- Show as much work as is needed to find out if the end behavior involves an asymptote, either horizontal or otherwise.
- Your graph must display exact roots, exactly positioned asymptotes of all types, correct behavior near each root, correct behavior near each asymptote, and correct end behavior.
- If you use a graphing calculator to help with any of this, “show your work” by sketching a picture of the calculator’s graph on your exam paper.

Section 4.1

- Be able to graph exponential functions.
- Be able to graph transformed exponential functions. Note that you must get one point and the asymptote exactly correct.
- Be able to solve for unknown constants in an exponential function.
- Be able to work exponential word problems, including compound interest problems.

Section 4.2

- Know the definition of logarithm:

  \[ \log_b x = y \iff b^y = x \]

- In standard base \( e \):

  \[ \ln x = y \iff e^y = x \]

- Be able to convert between exponential and logarithmic form.
- Be able to solve for unknown constants in a logarithmic function.
- Be able to graph logarithmic functions.
• Be able to graph transformed logarithmic functions. Note that you must get one point and the asymptote exactly correct.

• Be able to find the domain of a logarithmic function.

Section 4.3

• Know the algebraic laws of logarithms:

\[
\log_b(xy) = \log_b x + \log_b y
\]

\[
\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y
\]

\[
\log_b (x^n) = n \log_b x
\]

• Know what is NOT a law of logarithms. I.e.,

1. You CANNOT simplify \( \log(x + y) \)

2. You CANNOT simplify \( \frac{\log x}{\log y} \)

• Be able to expand a single logarithm until there are no products, quotients or exponents inside.

• Be able to collapse an expression into a single logarithm.

Section 4.4

• Know how to solve equations with the variable inside a logarithm. I.e., collapse to a single logarithm, then convert to exponential form.

• Know how to solve equations with the variable inside an exponential. I.e., isolate the exponential on one side of the equation, then take \( \ln \) of both sides.

• Be able to solve word problems involving exponential functions, including compound interest problems.

Section 4.5

• Be able to solve exponential growth/decay problems.

• Know that all such problems involve a physical constant called the “relative growth rate”.

• In many such problems, you will have to use some data to solve for this constant.